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Influence of the Dynamic Stress Intensity Factor on Cyclic Crack Propagation in Polymers

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Synopsis

A relationship of the form

$$d(2a)/dN = \alpha\beta\lambda^n$$

developed for the evaluation of cyclic crack propagation in tensile/tensile fatigue was used to investigate the effects of frequency on fatigue crack growth. In order to establish the correlation between stress intensity K and dK/dt at 21°C, dynamic fracture toughness tests were performed on a range of polymers. It was shown that, in general, fracture toughness increased with the strain rate applied. Consequently, a decreasing trend in the crack growth rate was observed in the fatigue tests performed at higher frequencies. The occurrence of other localized peaks of fracture toughness recorded at various temperatures and strain rates is described. The fractography of fatigue surfaces is also discussed.

INTRODUCTION

Fatigue criteria are important for the designer as well as for the stress analyst. For both purposes, such criteria need to be based on those mechanical properties which can be measured easily and yet accurately. It was recognized at an early stage of the analysis of fatigue failures that in general the considerations of crack propagation were more important for successful design of engineering structures than those of crack initiation. It was also realized that due to manufacturing limitations the presence of a sharp initial crack in the structure can not be excluded. Repeated loading is frequently encountered in engineering structures and fatigue failures are very common. Two mechanical processes then take place at the tip of a growing fatigue crack: intermittent crack propagation and strain hardening or softening; both have been studied by many investigators, with the aim to develop crack propagation laws covering a sufficiently wide range of cyclic conditions. Crack growth rate $d2a/dN$ was usually expressed as a function of alternating loads or stresses; these relationships were of limited application.

A number of criteria on the cyclic growth of cracks has been proposed and empirical expressions derived from tests on small, usually cylindrical, specimens indicated with some precision a proportionality between the crack growth per cycle, the alternating stress, and the crack length. The advent of Linear Elastic Fracture Mechanics opened a further avenue by providing a method of describing the situation at the tip of a static crack by a single parameter, the stress intensity factor K. It was soon realized that this concept might have much wider application in other situations than static failures, such as stress corrosion, creep, and, in particular, fatigue. Paris[1] showed that to the first approximation the cyclic crack growth is proportional to ΔK and suggested an expression

$$d2a/dN = C(\Delta K)^m \tag{1}$$

representing a linear relationship between the crack growth and ΔK in log–log coordinates of a slope m; C was a material constant. Subsequent research led to a number of refinements and consequent complications of the simple power law. Detailed studies of the crack growth curve, originally assumed to be a straight line, were carried out. It was noted that with many materials the complete curve was S-shaped. Only the middle part, limited by a lower and upper inflection point, could be described by eq. (1). Below the lower inflection point, the crack growth rates were found to be extremely low, as the growth occurred under a very low value of the stress intensity factor. Nonpropagating (dormant) cracks have been observed in this region; the cyclic stresses were below those corresponding to the minimum crack growth of the order of one lattice spacing per cycle. A different situation and a much faster than standard growth occurred above the upper inflection point, where the material failed rapidly.

Two conclusions are of interest in the present work. Firstly, fractographic observations now available suggest that a totally different fracture mode can be seen above the upper inflection point than is observed below. In this region of high stress amplitude, the crack growth occurs by simple formation. Below the inflection point, striations may be observed at regular intervals, their number corresponding with the number of cycles. Formation of striations in metals has been studied in detail; some information in the field of plastics is also available.[2] Secondly, it was noted that below the upper inflection point, the crack growth rates are not strongly influenced by the changes in yield strength or by fracture toughness. Evidence that crack growth is unaffected by both factors has been recently published;[3] information on plastics is scarce.

Recent investigations on a range of polymers covered the effects of the mean level and the amplitude of K as well as the effects of frequency on the fatigue crack growth.[4]. On the basis of these results, a new relationship for crack propagation discussed below was proposed in the form

$$d(2a)/dN = f(K^2)^m. \tag{2}$$

The mechanism of this cycle-dependent crack propagation can be described for many metals and polymers in terms of highly localized intense plastic straining; it was discussed in detail by Laird.[4] The accompanying periodic blunting and sharpening of the crack are connected with the formation of striations. The local conditions at the crack tip are influenced by the strain rate dependence of the yield, and thus of fracture toughness K.

The influence of frequency observed in fatigue tests emphasizes that strain rate sensitivity will play an important part in the crack propagation process. In addition to measuring the critical static values of K, it is therefore necessary to know the spectrum of K for the investigated range of dK/dt, corresponding to the applied frequency of the fatigue process as a function of temperature. As this problem would require an excessively large number of tests, simplified methods of static, quasistatic and dynamic K measurements such as described in the ASTM specifications[4] were used. It is known that many plastic materials show a considerable decrease of the cyclic crack propagation rate with increasing frequency; this behavior has not yet been satisfactorily explained. This paper suggests a new approach to the problem of cyclic crack growth by including in the analysis the influence of the critical value of K_I at the appropriate loading rate. Fatigue results obtained on three plastic materials are discussed. While static fracture toughness data on many materials are available, there is a definite shortage of comparative results derived from dynamic tests.

The results from tests performed on a range of materials showed that the strain rate sensitivity α can be expressed as a ratio of $K_{\text{static}}/K_{\text{dynamic}}$. Materials with ratio $\alpha > 1$ are positively sensitive (e.g., some mild steels), whereas ratio $\alpha < 1$ indicates a negative sensitivity (e.g., plastics at certain temperatures). When this ratio is unity, the material will not show any definite strain rate sensitivity (Al alloys, high-strength steels).

The proportionality between the flow stress and fracture toughness should be recalled again. It is known that in many materials the flow stress has a positive strain rate sensitivity. Under those conditions, and with no change observed in the fracture mode during the fracture process, the fracture toughness K will increase with increasing strain rate. There is considerable data available on the strain rate dependence of the flow stress for a variety of metals and polymers. However, results from higher strain rate tests are scarce. In this paper the experimental evidence is presented showing that in the negatively sensitive materials the crack propagation rate decreases with increasing sensitivity. It is also suggested, that conversely, positively sensitive materials will show an increased crack propagation with increasing frequency, while the third group is not frequency sensitive.

EXPERIMENTAL PROCEDURE AND RESULTS

Three plastic materials were chosen for the fatigue tests. Firstly, cast sheets of poly(methyl methacrylate) (PMMA), as supplied by ICI, were cut into specimens sized $13^1/_2 \times 15 \times {}^1/_4$ in. Extruded sheets of polycarbonate (PC) bisphenol A (Macrolon), made by Bayer, were cut into slightly smaller specimens, $7 \times 12 \times {}^1/_4$ in. Similarly, PVC specimens were of the same size as PC, but ${}^1/_2$ in. thick; this material was supplied by

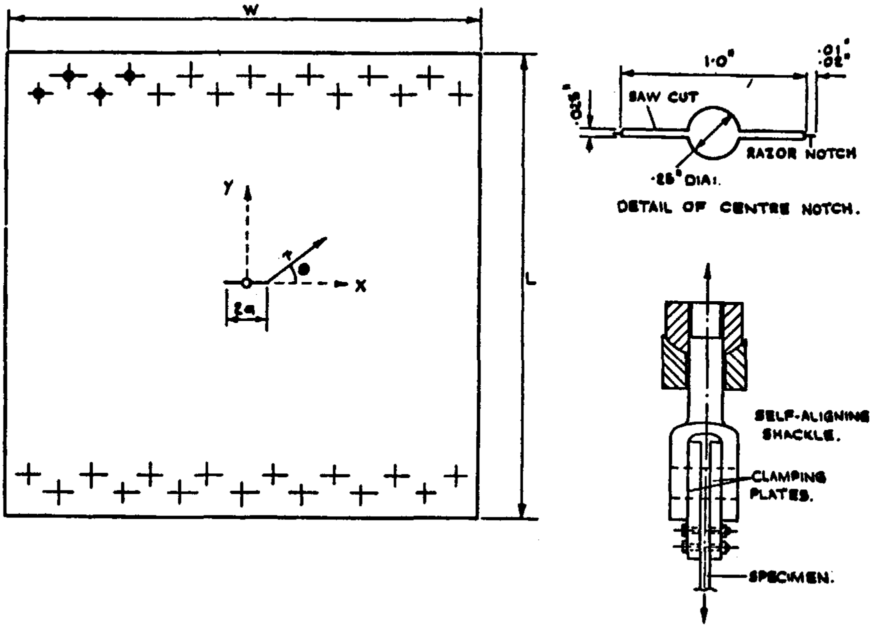

Fig. 1. Fatigue specimen and grips.

BP (United Kingdom). The specimens were not annealed, but had been stored in the laboratory for a minimum period of three months. A 1/4-in. hole, drilled in the center of each plate (Fig. 1), was provided with a slot (parallel to the direction of extrusion), which in turn was extended into a sharp crack with a razor blade. The sharpness and regularity of the starter crack was found to be of importance; uneven or skewed notches were not used. The influence of a sharp starter on static or cyclic test results is well known.[5]

All specimens were tested in a Dowty electrohydraulic fatigue testing machine, 12,000 lb-ft capacity, capable of cyclic frequencies up to 6000 cyc/min. The detailed manufacture of specimens and the notch sharpening procedure, as well as the test method used, are described elsewhere.[4] In general, fatigue tests, consisting of wholly tensile cycles between closely controlled limits of stress intensity factor K_{max} and K_{min} at a range of frequencies, were performed in standard laboratory conditions of 21°C and 50% R.H. Appropriate load limits were manually adjusted while each test was in progress and the crack growth monitored by a cathetometer. Only specimens with symmetrically propagating cracks, of total length $2a$ not more than 0.4 W, were included in the analysis. Subsequently, these specimens, as well as others with shorter cracks, were tested statically. In order to ascertain any possible temperature increase during the crack growth at higher frequencies, ultimately leading to a thermal failure, a temperature sensor was placed close to the crack path in the preliminary tests; no noticeable change in the surface temperature was noted even at

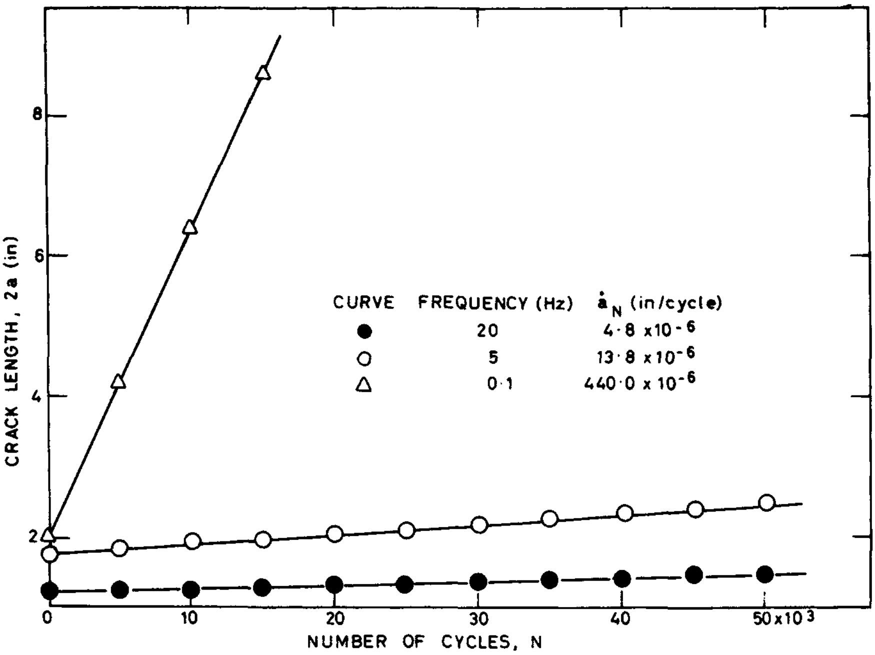

Fig. 2. PMMA: crack length vs. cycles at $K_{\max} = 750$, $K_{\min} = 250$ psi$\sqrt{i}$.

the highest frequency of 20 Hz. A small increase of a few degrees centigrade above the testing temperature was, however, observed in comparable tests on polycarbonate.[6]

All three materials were tested under conditions of constant ΔK. This method was considered more suitable for the present work than fatigue tests conducted under "constant load." In the load cycling tests, the results might be influenced by cyclic creep, the amount of which would represent additional strain applied on the specimen. The upper boundary of ΔK range was limited by the level of $K_{\max}$ at which the amount of dynamic creep would become significant and might influence the crack growth process; such a maximum level for PMMA was 750 psi$\sqrt{i}$. The three frequency levels used were 0.1, 5, and 20 Hz; this modest range, spanning only about two orders of magnitude, was however sufficiently wide to show the influence of frequency on crack propagation and was convenient for precise monitoring of the crack growth in the isothermal conditions of the tests.

Figure 2 shows the results of cyclic crack propagation tests on PMMA conducted between $K_{\max}$ of 750 and $K_{\min}$ of 250 psi$\sqrt{i}$ at a range of frequencies. It will be noticed that in this case the crack growth $\dot{a}_N$ increased 100 times while frequency decreased from 20 Hz to 0.1 Hz. Other lower-frequency tests indicated an increasing influence of the mean stress intensity factor K_m on the crack growth, which is discussed in more detail in ref. 7.

Measurements performed in the second series of cyclic tests on polycarbonate showed again a decreasing crack growth rate with increasing

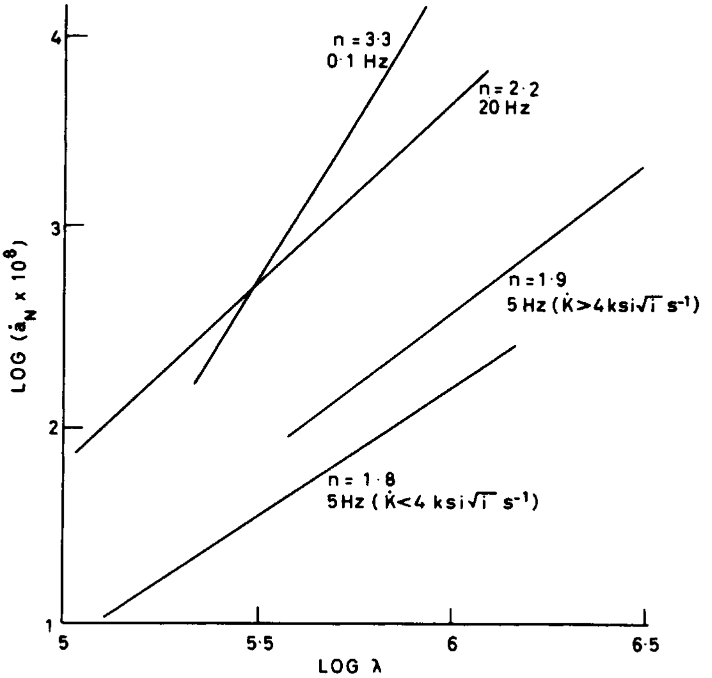

Fig. 3. Polycarbonate: 21°C effect of frequency on exponent $n(\dot{a}_N = \beta\lambda^n)$.

frequency, the behavior being analogous to that of PMMA plotted in Figure 2. Subsequent analysis of the tests on PC pointed toward a significant influence of the mean K_m. However, with further increase in ΔK, the effect of K_m appeared to be diminishing and finally ceased at $\dot{K}$ equal to 4×10^3 psi$\sqrt{i}$ s^{-1}, while the influence of ΔK on the crack growth became preponderant. (All subsequent values of K are quoted in psi$\sqrt{i}$, and $\dot{K}$ in psi$\sqrt{i}$ s^{-1}, where $\dot{K} = dK/dt$.)

The effect of frequency was studied in a large number of tests performed at constant ΔK. Figure 3 shows such a relationship between $\dot{a}$ and ΔK for the tests with $\dot{K}$ greater than 4×10^3. It may be observed that the crack growth increases with higher frequencies. The behavior below $\dot{K}$ equal to 4×10^3 was unexpected; it was found that in this regime the crack propagated faster with decreasing frequencies. The results of these tests showing the slowing down followed by a subsequent increase in the crack propagation rate are plotted against frequency in Figure 4. Comparative results for different ranges of ΔK are also included. Minimum crack growth rate appears to be reached at around 5 Hz for tests with ΔK of 300 psi$\sqrt{i}$. Because of the dependence of the transition point on the strain rate $\dot{K}$, a shift of the minimum crack growth toward lower frequencies in order to counterbalance the influence of increasing ΔK can be expected; this shift was observed in the tests at ΔK equal to 500 psi$\sqrt{i}$. It is probable that this transition region, which is important in the crack propaga-

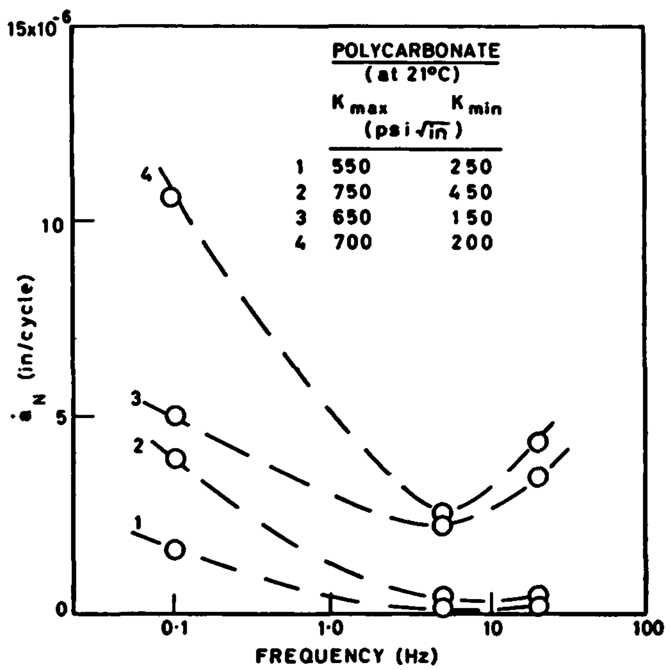

Fig. 4. Polycarbonate: frequency vs. $\dot{a}$.

tion process, is connected with the maximum of the relaxation losses observed in vibration experiments.

The analysis of the above results provided additional evidence of the influence of strain rate on the value of the exponent n in eq. (2). In the region below $\dot{K}$ equal to 4×10^3, the value of n decreased from 3.33 at 0.1 Hz to 1.8 at 5 Hz, while at $\dot{K}$ values above 4×10^3, the exponent n increased to 2.2 (tests at 20 Hz); this suggests again a minimum of n around 5 Hz.

A larger proportion of PVC specimens, although of the same size as previously tested PC test pieces, were provided only with one side notch (SEN type). These results correlated well with comparative tests on CN specimens. Typical crack growth curves for various frequencies are shown in Figure 5. It was observed that the increase in frequency again resulted in a decrease in the cyclic growth rate, Figure 6. The change of the growth rate due to higher ΔK values and higher frequencies was less prominent than that recorded in slow cycling. This change of the growth rate is associated with two different modes of crack propagation occurring below and above the upper inflection point of the $\dot{a}$ versus ΔK curve. In the fatigue tests performed at low and medium amplitudes ($K \leq 2.5$ ksi$\sqrt{i}$) striations similar in appearance to those reported on PMMA[2] were observed; they were straight, evenly spaced, and parallel with the base of the notch (some were slightly bowed out in the direction of crack propagation) and showed a one-to-one correspondence with the number of cycles. A

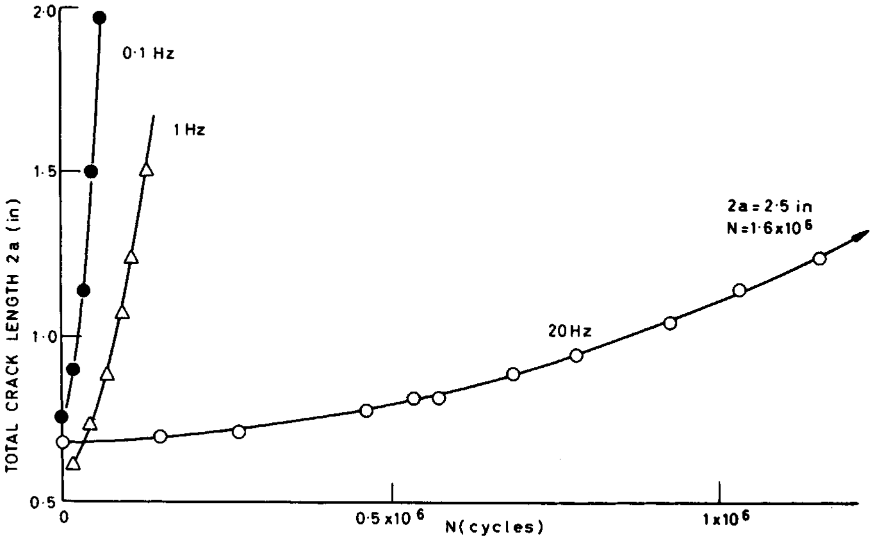

Fig. 5. PVC: load cycling at 21°C; P_{max} = 2100 lb-ft.

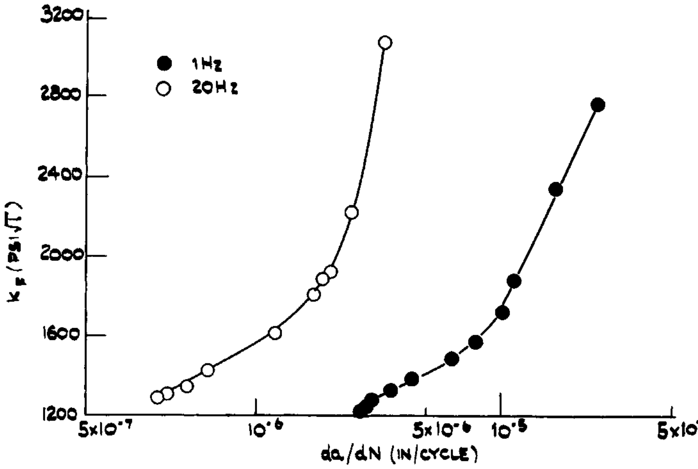

Fig. 6. PVC: K_F vs. da/dN, 21°C.

similar correlation in the fatigue of polycarbonate and polyethylene was observed by McEvily et al.[8]

This slow crack growth rate corresponds to the conditions prevailing in the regime below the inflection point. However, when the tests were performed at higher amplitudes ($K \geq 2.5$ ksi$\sqrt{i}$), increased plastic deformation was noticed and the "brittle" fracture mode gradually appeared to change into tearing. It was also found that in this regime (above the upper inflection point), not every application of load produced an incremental crack extension, i.e., the number of striation markings was less than the number of cycles, the ratio being approx. 1:2. It is interesting to note that the fatigue tests on PVC recently reported[9] have also shown an intermittent crack propagation between 200 and 1000 load cycles.

Fractographic examinations of fatigued surfaces produced at other ΔK values, i.e., below the lower transition point or in the instability region, are not described here as they have been discussed elsewhere for PMMA,[2] and reports on PC and PVC are now in the course of preparation. Suffice it to add that in the present tests, specimens of adequate thickness were used, thus the cracks propagated under plane strain conditions and large areas covered with striation markings were observed. However, it is not clear why the remaining parts of the fatigued surfaces did not show any sign of striations; further work on this subject is necessary. It is known that, in general, fatigue striations are not found in the region of plane stress. The present as well as other investigations[2,4,5] suggest that the fatigue crack propagation studied on a range of polymers is similar to the process observed in metals. It should be added that during cyclic crack propagation, no crazing phenomenon was noticed and the cracks progressed by a craze-free growth.

DISCUSSION

Application of LEFM in the field of fatigue originated in the work of Paris.[1] The now well-known relationship demonstrated a correlation of the crack growth rate with the K range during tensile cycling, limited between $K_{\max}$ and zero load ($K_{\min}$). The simplicity of this approach indicated the primary importance of the cyclic range ($K_{\max} - K_{\min}$), while the influence of mean K_m on the crack growth rate was considered to be only of second order. While there is very little evidence of the K_m influence on the growth rate available in the plastics field, seemingly contradictory results have been reported for metals. Particularly strong evidence on the importance of mean stress which is proportional to K_m in fatigue tests on mild steel bars has been recently presented.[10] Similar results have been obtained on cast steels and some Al alloys. On the other hand, the work on high-strength steels and Al alloys indicated that K_m was of minor importance as compared with ΔK. It was realized that the situation with metals may be exceedingly complex, and in view of the limited amount of dynamic toughness data at present available, it was thought that a study of plastics might usefully serve as a model for the understanding of cyclic crack growth in metals.

A large number of modifications to the basic crack growth law, eq. (1), have been proposed and are now in use for solving design problems. Some of these modifications account, more or less successfully, for the influence of either K_m or of the stress ratio R (equal $S_{\min}/S_{\max}$), which may be expressed as $K_{\min}/K_{\max}$, and have been evaluated in ref. 5b. Based on an extensive test program covering a range of plastics at various frequencies and levels of $K_{\max}$ and $K_{\min}$, a greatly simplified relationship has been proposed[4] taking the form of

$$d2a/dN = \beta\lambda^n \tag{3}$$

where

$$\lambda = K_{\max}^2 - K_{\min}^2 = 2\Delta K K_m, \tag{4}$$

n = numerical factor varying with testing conditions and environment (thus it may be expected to change with temperature or humidity), and β = factor depending on material and loading conditions. For cycling at $2K_m = K_{\max}$, n becomes $m/2$, and eq. (3) reduces into Paris's relationship

$$\dot{a}_N = C(K_{\max}^2)^n = C\Delta K^m. \tag{1}$$

By including the stress ratio R into eq. (4), we obtain

$$\lambda = (1 - R^2)K_{\max}^2 \tag{5}$$

and eq. (1) may be expressed as

$$\dot{a}_N = \beta(1 - R^2)^n(K_m^2)^n. \tag{6}$$

Note that although the values of $S_{\max}$ and $S_{\min}$ remain constant during the fatigue process, the increasing crack size does not change the stress ratio R, whereas the K values are increasing.

A method of computation of the total cyclic life accounting for the increasing K and frequency, thus permitting the study of the crack propagation process from the initial flaw, was suggested.[11] It was shown that the influence of frequency on the crack growth in plastics was distinctly greater than that observed in the behavior of metals, where the effect of changes of frequency were often very small. In particular, those findings were supported by the test results such as on PC, Figure 4.

During the subcritical period of fatigue crack propagation, the crack length as well as $K_{\max}$ increases until $K_{\max}$ attains the critical value of K_{1C}, when the situation becomes unstable and the crack extends catastrophically. In order to establish the law for a propagating crack, it is therefore necessary to know not only the ΔK values but also the appropriate value of K_{1C} for the loading rate and conditions prevailing at the moment of crack growth. It was realized that, in general, the value of K_{1C} will change with temperature, but the change due to the testing environment, and in particular due to the strain rate, was not well understood. A convenient expression for the strain rate is

$$\dot{K} = dK/dt = (K_{\max} - K_{\min})/{}^1/_2 \text{ periodic time.} \tag{7}$$

While there is a large amount of K_{1C}-versus-temperature data available for many materials, there exists at present only limited information on the relationship of K_{1C} versus $\dot{K}$ at constant temperature. Even the basic diagrams for the three plastics discussed here are far from complete. Undoubtedly, one of the materials attracting wide interest is PMMA. The graph Figure 7 shows the relationship of K_{1C} versus crack speed (strain rate) at 21°C; the curve, compiled from a large number of tests, is fully described in ref. 12. While the toughness values at very low crack speeds are nearly constant, a rapid increase occurs within a comparatively narrow region and

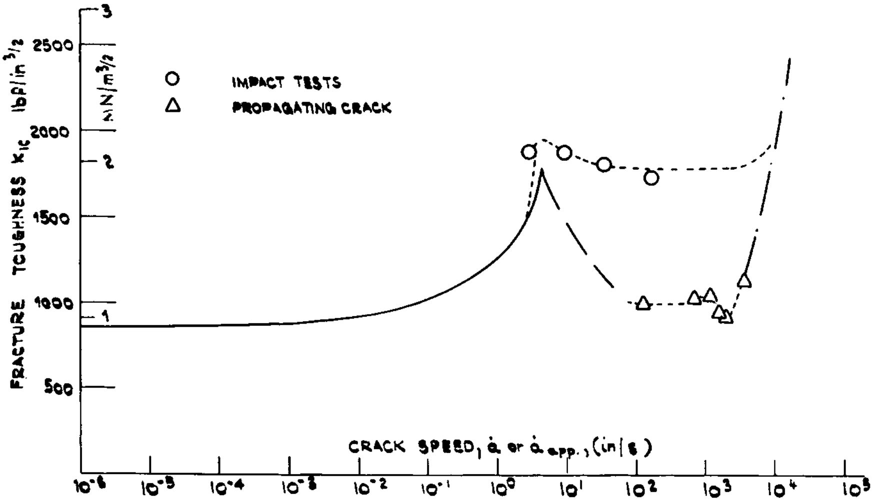

Fig. 7. PMMA: fracture toughness vs. crack speed at 21°C.

terminates as a singularity point at a speed of 1 in./s corresponding to $\dot{K}$ = 3.3 × 10^4. Below this critical point, the fracture mode was a stable, slow growth, whereas above it, the fracture became unstable, fast, and had a brittle appearance with total absence of a slow growth region. At higher crack speeds, the toughness values remained very much constant and slightly below the singularity peak, until again at very high crack speeds, above 10^4 in./s, corresponding to $\dot{K}$ equal to 10^8, K_{IC} would increase considerably. No attempt was made to perform fatigue tests in the unstable crack region delineated in Figure 7, particularly because of the difficulties connected with the constant-temperature condition at the crack tip. Also, it was realized that in this regime the growth of a crack, once initiated, may not stop, but develop into a propagating crack the toughness value for which was distinctly below that connected with the initiation. Cyclic tests were characterized by the K-value below the critical K_{IC}, hence the crack extension was stable. Most importantly, however, the critical toughness value was found to increase monotonically with the increasing frequency. It was therefore concluded that under these conditions the crack growth rate would decrease, cf. Figure 2. While this behavior may be found of general validity, a somewhat more complicated growth could be expected at other temperatures where different and stronger mechanisms may take place. A greatly increased toughness at lower temperatures may serve as an illustration. At a very limited region, around −80°C, toughness values in standard impact tests were found to increase to approximately 3400 psi$\sqrt{i}$ (i.e., by 70%); similar localized increases at lower strain rates were observed at still lower temperatures. It could be expected that under such conditions, the cyclic crack growth rate would be reduced yet more. Tests to verify this are now in preparation.

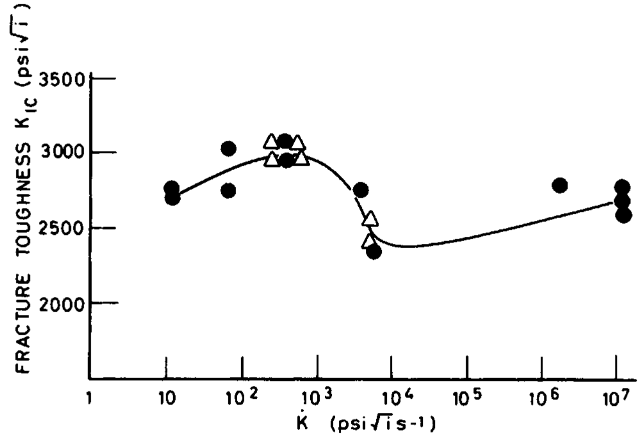

Fig. 8. PC: K_{IC} vs. $\dot{K}$ at 21°C. (Δ) Key et al.[13]

While PMMA may provide an excellent example of a continuously increasing K_{IC} with the strain rate, other and more complicated behavior was observed in the tests with PC. Here, in the region of slow strain rates, Figure 8, K_{IC} was found to be slowly increasing and the maximum of 3100 psi$\sqrt{i}$ was reached at $\dot{K}$ equal to 10^3. Thereafter, the toughness values showed a steep decrease to 2400, followed by subsequent gradual increase towards the fast strain rates of the order 10^7. It could be speculated that the observed toughness peak may bear a close relationship with the maximum value of K_{IC} recorded in PMMA at -80°C. However, it is not at present possible to explain the K_{IC} peak on the basis of molecular kinetics, as no sufficiently detailed relaxation studies are yet available and also more toughness data are needed. While an additional testing program is in progress, it is encouraging to note that the only other published results known to the author[13] closely correlates with the maximum and minimum toughness observed. It is now apparent that the K_{IC} maximum coincides with the minimum crack growth rate under same testing conditions, i.e., at $\dot{K} = 4 \times 10^3$, Figure 4. The increasing crack growth rate $\dot{a}$ below as well as above the critical $\dot{K}$ may be correlated with the decrease of K_{IC} shown in Figure 8. If isothermal testing conditions at much higher frequencies ($\dot{K}$ equal to 10^4 and higher) could be achieved, a further slowing-down af the crack propagation process would take place.

The final attempt concerned the evidence necessary to explain the decreasing crack growth in the fatigue process of PVC. Four series of three-point bend tests covering a wide range of cross-head speeds were performed: 209 in./s (standard impact), 36 in./s, 0.02 in./s, and 3×10^{-5} in./s (standard slow bend). Recent fracture toughness tests described in detail in ref. 14 indicated that the K_{IC} impact values (corresponding to $\dot{K}$ equal to 2×10^7) in air and at room temperature were somewhat below those measured in the standard slow bend tests, Figure 9. These results would suggest that under comparable testing conditions, the crack propagation rate would be highest

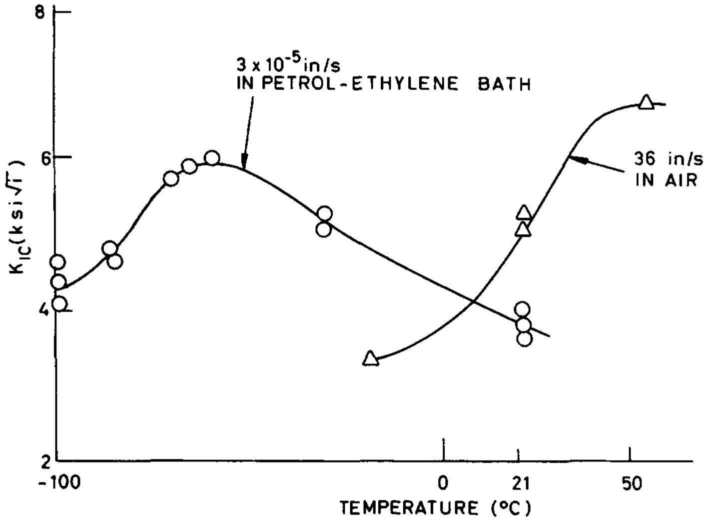

Fig. 9. PVC: K_{IC} vs. temperature influence of strain rate.

at the cyclic frequency corresponding to the strain rate of a standard impact test. However, it was realized that the slow bend values were only apparent ones due to the excessive amount of plasticity occurring during the test, while fracture surfaces in impact tests were sufficiently brittle.[5] Additional impact tests were therefore conducted at lower $\dot{K}$ of 1×10^6 and showed a definite increase of toughness. Fracture toughness data obtained in the final series of tests at $\dot{K}$ equal to 1×10^3 were only marginally higher than the results of slow bend tests ($\dot{K} = 2$), but still below the intermediate tests. As expected, the degree of brittleness increased with increasing crosshead speed. As the value of $\dot{K}$ applied in the fatigue tests fall between fast (0.02 in./s) bend tests and low angle (36 in./s) impact tests, it can be seen that the K_{IC} values from the monotonic tests are indeed applicable and may be used in the analysis of the cyclic crack growth data. It is appreciated that additional fatigue tests, in particular at a range of temperatures, would be found of help in the generalization of the described cyclic behavior.

The reduction of the crack growth rate with increasing frequency observed in PVC resembled closely the behavior of PMMA and PC, Figure 5. Further, two factors connected with plastic zone size and fracture appearance were of importance in the crack growth evaluation. Apart from the K_{IC} value which was found increasing with the strain rate, Figure 9, the yield stress was also affected by the changes of frequency. The increase in the value of σ_y between tests performed at crosshead speeds of 0.02 in./s and 36 in./s amounted to 40%. If the strain rate sensitivity of the yield stress were higher than that of fracture toughness, plastic zone size would decrease with increasing frequency and the crack would ultimately propagate in a brittle fashion. A large plastic zone would be formed at lower frequencies with a consequent development of ductile failure.

CONCLUSIONS

1. Influence of frequency on the crack propagation rate in cyclic tension tests performed on a range of plastics was investigated. The Linear Elastic Fracture Mechanics approach was applied and crack growth evaluated in terms of the stress intensity factor.
2. Critical fracture toughness at a strain rate corresponding to the frequencies applied in the fatigue tests provides a suitable indication of the cyclic crack propagation rate. With increasing fracture toughness, fatigue crack propagation rate will decrease.
3. In general, plastic materials so far investigated at 21°C show an increasing dynamic fracture toughness with increasing strain rate. However, in limited regimes of strain rate fracture toughness may decrease.
4. The knowledge of the fracture toughness spectrum in appropriate testing conditions is a prerequisite for the reliable prediction of fatigue crack growth.

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